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ABSTRACT

Multiple access (MA) in UWB communication is an area of active research. In this paper we introduce and study the performance of a new MA scheme in the context of multiple transmitted-reference short duration (nsec) chirp pulses in the presence of additive white Gaussian noise (AWGN). The transmitted-reference (T-R) receiver is extended using multiple orthogonal pulses. The proposed UWB receiver samples the receiver autocorrelation function (ACF) at both *zero-* and *non-zero* lags, thus sampling and matching the shape of ACFs rather than just the shape of the received pulses. Sampling of non-zero ACF lags is a significant new approach. The scheme proposed in this paper is a step towards combining the multi-pulse approach and T-R modulation in a multiple access ultra wideband (MA-UWB) communications system. Improved bit error rate performance over a conventional *zero-lag* receiver (i.e. energy detection receiver) is demonstrated by simulation. Analytical expressions for the system BER are also derived and confirmed through simulations for the system.

1. INTRODUCTION

Ultra-wideband (UWB) technology has received a significant attention from communications industry since the Federal Communications Commission (FCC) rulings in February 2002. According to FCC, UWB signals have fractional bandwidth (B_f) of 20% or larger at –10 dB cut-off frequencies [1]. Unlike, traditional communication systems that use continuous wave sinusoids, UWB systems use carrierless, short duration (picosec to nanosec) pulses to transmit and receive information. The short duration of UWB pulses spreads their energy across a wide range of frequencies from near DC to several Gigahertz. This large bandwidth provides high capacity, robustness to jamming and low probability of detection properties for UWB communication systems. Furthermore, UWB technology has the potential of delivering large amount of data with low power spectral density and is proved to be useful in short range, high data rate applications [2]. Such applications require that several transmitters coexist in the covered area. Therefore, proper multiple access techniques are needed to perform the channelization for multiple users. In a multiple access UWB communications system, users transmit information independently and concurrently over a shared channel. The received signal is therefore a superposition of all user signals with added channel noise. There has been extensive research in separating multiple users in a multiple access UWB system using TDMA or CDMA Pulse

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Amplitude Modulation (PAM) and Pulse Position Modulation (PPM) techniques [4, 5, 6 and 7]. PAM modulation encodes the data bits based on different levels of power (amplitude) in short duration pulses. In PPM modulation, signals are pseudo randomly encoded based on the position of transmitted pulse trains by shifting the pulses in a predefined window in time. PPM transmitted signals are usually demodulated and recovered with template matching at the receiver. Conventionally, both of these modulation techniques use a single pulse shape to transmit and receive data for all users. One concern with using the same pulse shape for all channels is that multiple access interference (MAI) rises with the increase in number of users. This is due to increased cross-correlation between similar pulses from various channels, raising thus the noise floor in such systems. To overcome high cross-correlation between similar pulses, Ghavami and Kohono in [10] have suggested the pulse shaping modulation using Hermite-based orthogonal pulses. Using multiple pulses with low cross-correlation in multiple access systems reduces the MAI effect in multi-user environments.

In this paper we extend the previous work on UWB delay modulation technique [8] and transmitted-reference (T-R) receivers [9] by using multiple orthogonal pulses. Authors of [8,9] proposed to use a symbol that consists of a pair of pulses (doublets) separated by a unique delay to represent each bit. The first pulse is fixed and the second pulse is modulated by data using opposite polarities to represent one or zero. This method has the advantage of sending the same pulse twice through an unknown channel where both pulses are distorted the same way and detection becomes easier with an autocorrelation receiver. However their method uses a single UWB pulse shape for all channels. The scheme proposed in this paper uses multiple orthogonal pulses for a UWB system and is a step towards combining the multi-pulse approach and T-R modulation in a multiple access ultra wideband (MA-UWB) communications system. This paper studies the performance of a multi- pulse multi-delay (MPMD) modulation scheme [3] both analytically and empirically. The pulses used in this paper are mutually orthogonal chirp pulses. The idea of using chirp signals for communications was first introduced by Winkler [10]. Chirp signals perform well at the presence of fading due to multipath and have been used in high frequency data transmission applications extensively [12]. El-Khamy, et al proposed the use of chirp signals for efficient multiple access communications in [13].

Section 2 describes the basic parameters for the waveform and transmitter design for MPMD modulation scheme. Section 3 describes the receiver design and its demodulation technique. Section 4 provides a comprehensive analysis of BER performance of a MPMD receiver followed by conclusions in section 5. Detailed derivation of systems' SNR is available in Appendix A.

2. MPMD TRANSMITTER

In this model the data bits from multiple users are transmitted using multiple mutually orthogonal pulses (unique to each user) modulated with T-R scheme. The basic building block for a symbol in T-R modulation consists of two similar pulses separated by a delay (D). The first pulse is fixed and called reference pulse (ref) and the second pulse is modulated with data called data pulse (data). The data pulse modulation scheme is based on the polarity of the reference pulse. For instance a reference and a data pulse of the same polarity designates a binary value of 1, while a data pulse opposite in polarity with reference pulse designates a binary value of 0. Assuming a uniform symbol period, the general signal model of a MPMD system with N users be expressed as

$$S_{n,m}(t)=\sum_{n=1}^{N}\sum_{m=1}^{M}\sqrt{E_{p,n}}[P_n(t-(m-1)T-(-1)^{b_m(n)}.P_n(t-(m-1)T-D_n)] \quad (1)$$

Where

N = Number of users
M = Number of transmitted bits
$E_{p,n}$ = n^{th} user's signal energy (normalized for all users)
$P_n(t)$ = n^{th} user's UWB pulse
$b_m^{(n)}$ = $[b_1^{(n)}, \ldots, b_M^{(n)}]$ n^{th} user's m^{th} data bit, $b_m^{(n)} \in [0,1]$
D_n = n^{th} user's delay
T = Pulse repetition period

The UWB pulses used in this work are short duration chirp pulses with different start and end frequencies. Chirp pulses that do not overlap in frequency band are theoretically uncorrelated with each other. The orthogonal behavior of pulses can be shown by their NxN correlation coefficients matrix given by

$$\Lambda_{Chirp}=\begin{bmatrix} 1 & r_{12} & . & . & . & r_{1N} \\ r_{21} & 1 & . & . & . & r_{2N} \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & . \\ r_{N1} & r_{2N} & . & . & . & 1 \end{bmatrix} \quad (2)$$

Where

$$r_{ij}=\begin{cases} 1 & i=j \\ \approx 0 & i\neq j \end{cases}$$

As shown, the cross-correlation terms of any two uncorrelated pulses are much smaller than the autocorrelation coefficients for each pulse. Due to their orthogonal behavior, these pulses can be separated using autocorrelation technique shown below.

$$S(t)=S_1(t)+S_2(t)+\ldots+S_N(t) \quad (3)$$

$$S_1(t)\oplus S(t)=\underbrace{S_1(t)\oplus S_1(t)}_{d(t)}+\underbrace{\cancel{S_1(t)\oplus S_2(t)}}_{0}+\ldots+\underbrace{\cancel{S_1(t)\oplus S_N(t)}}_{0} \quad (4)$$

3. MPMD RECEIVER

The MPMD receiver uses the following facts to recover the UWB pulses from various channel distortions due to multipath, fading, and noise.

1) The shape of UWB pulses can be considerably degraded due to channel distortions, while the shape of their ACF is preserved at the receiver.
2) Wideband pulses usually have autocorrelation functions with strong side lobes.
3) Information stored in ACF side lobes is available for free and can provide significant performance improvement in detection of multiple pulse systems.

MPMD receiver samples the ACF of each user's pulse at both *zero*- and *non-zero* lags and matches them to the corresponding samples taken at transmitter rather than sampling and matching the pulse shape. The input signal to the proposed receiver is given by

$$R_{n,m}(t) = S_{n,m}(t) + w(t) \tag{5}$$

Where $S_{n,m}(t)$ is the combined signal for N users and M bits from (1) and $w(t)$ is AWGN with zero mean and two-sided power spectral density $N_0/2$.

The proposed demodulation scheme has two steps. Step one provides estimates for multiple sampled values of the received signal's ACF. These estimates are achieved from multiplying the received signal by its multiple delayed versions. The second step involves matching the estimated values from previous step to the sampled values of ACFs of original pulses. Note matching is performed over the second order statistics, such as the ACF, and not on the signal shape, as is conventionally performed in classical matched filters. Fig. 1 represents the MPMD receiver block diagram.

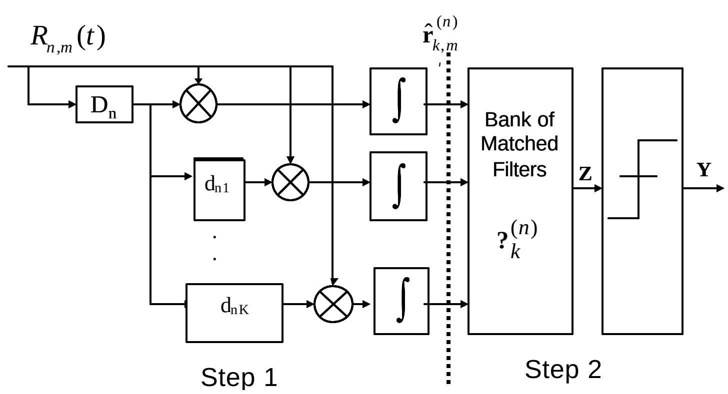


Figure 1: UWB Receiver Block Diagram

As shown in Fig.1 the receiver employs autocorrelation technique by multiplying the received signal with multiple delayed versions of itself and integrating over a finite time. Then a bank of matched filters matched to ACF samples of original pulses at transmit stage for each user ($\mathbf{?}_k^{(n)}$) followed by a hard decision block separates each channel. Note that D_n and d_n are unique for each receiver channel. The output of the receiver is

$$\mathbf{Y} = \mathrm{sgn}(\underbrace{\mathbf{?}_k^{(n)} . \hat{\mathbf{r}}_{k,m}^{(n)}}_{\mathbf{Z}}) \tag{6}$$

Where

$$\mathbf{?}_k^{(n)} = \left[R_{p_n p_n}^{(n)}(1) \quad . \quad . \quad . \quad R_{p_n p_n}^{(n)}(K) \right] \tag{7}$$

$$\hat{\mathbf{r}}_{k,m}^{(n)} = \begin{bmatrix} \hat{r}_{(1,1)}^{(n)} & . & . & . & \hat{r}_{(1,m)}^{(n)} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ \hat{r}_{(K,1)}^{(n)} & . & . & . & \hat{r}_{(K,m)}^{(n)} \end{bmatrix} \tag{8}$$

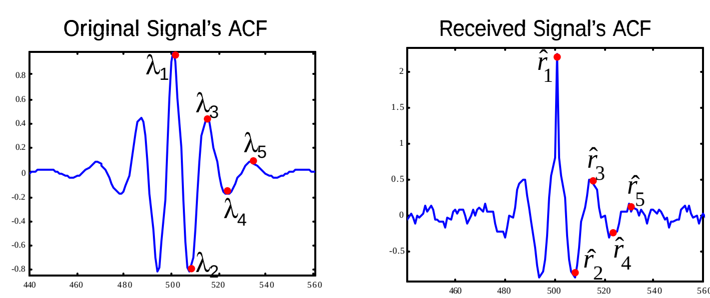
$\mathbf{?}_k^{(n)}$ denotes the normalized autocorrelation vector of nth user's transmitted pulse for K sampling points (lags) and $\hat{\mathbf{r}}_{k,m}^{(n)}$ is the autocorrelation matrix of nth user's mth bit for K lags which represents the estimated ACF sampled points at the receiver. Each row of the matrix in (8) represents the autocorrelation between the received signal and its delayed version based on various delays as shown in (9).

$$\hat{r}_{k,m}^{n} = \int S_n(t).S_n(t - \Delta_n)dt \tag{9}$$

Where Δ_n is the total delay as

$$\Delta_n = D_n + d_{nk} \tag{10}$$

D_n represents the nth users main delay that provides lag *zero* in the ACF, and d_{nk} denotes the offset from its main delay or lag k in ACF. Delaying the received signal by D_n causes the "ref" pulse to align with the "data" pulse in each symbol and their product decodes the symbols for received data bits by capturing the energy in lag *zero* of the generated autocorrelation function. The product of aligned pulses generates positive values when the symbol represents a binary 1, and negative values for a binary 0. Integrating the product values, samples the ACF at *lag zero*. Further delaying the received signal by multiple offsets (d_{nk}) added to the main delay (D_n) and multiplying with its un-delayed version, samples the autocorrelation function in *non-zero* lags after integration. The sampled points are estimates of the received signals' ACF. These estimated values ($\hat{\mathbf{r}}_{k,m}^{(n)}$) are then matched to the original pulses sampled values of autocorrelation function ($\mathbf{?}_k^{(n)}$) and provide a more accurate decoding of the received symbols. Fig. 2 illustrates matching of ACF samples in transmitted and received pulses.



$\mathbf{Z} = \mathbf{?}_k^{(n)T} . \hat{\mathbf{r}}_{k,m}^{(n)}$ Where: n=1, 2, ..., N (# of users)

k=1,2,...,K (# of lags)

m=1,2,...,M (# of bits)

Figure 2: Matching the Shape of ACFs for a Received and Transmitted Pulse

Assuming the system model is analyzed based on free space propagation conditions with AWGN and perfect synchronization for the user of interest, the output of the receiver is given by

$$\hat{r}_{m,k} = \int_0^{T_{in}} [P_n(t-(m-1)T)-(-1)^{b_m^{(n)}}.P_n(t-(m-1)T-\Delta_n)+w(t)].$$
$$[P_n(t-\Delta_n-(m-1)T)-(-1)^{b_m^{(n)}}.P_n(t-(m-1)T-2\Delta_n)+w(t-\Delta_n)]dt \qquad (11)$$

The integration time (T_{in}) plays an important role in the performance of the system. If (T_{in}) is smaller than the pulse width (T_p), only a fraction of pulse energy is captured and if (T_{in}) is larger than (T_p), noise terms accumulate with no additional signal energy and cause a decrease in SNR. Therefore, the integration interval should be equal to pulse width (T_p) in order to gather most of the signal energy. For simplicity and without the loss of generality, assume the received signal of a single user system sending its first bit, $b_1^{(1)} = 1$ as

$$\hat{r}_k = \int_0^{T_p} [P(t)+P(t-\Delta_n)+w(t)].$$
$$[P(t-\Delta_n)+P(t-2\Delta_n)+w(t-\Delta_n)]dt \qquad (12)$$

Expansion of (12) and collecting the related terms results in the following set of integrals

$$\hat{r}_k = \int_0^{T_p} [P(t-\Delta_n).P(t-\Delta_n)]dt + \int_0^{T_p} [P(t).P(t-\Delta_n)]+[P(t).P(t-2\Delta_n]+$$
$$[P(t-\Delta_n).P(t-2\Delta_n)]dt + \int_0^{T_p} [P(t).w(t-\Delta_n)]+[P(t-\Delta_n).w(t-\Delta_n)]+ \qquad (13)$$
$$[w(t).P(t-\Delta_n)]+[w(t).P(t-2\Delta_n)]dt + \int_0^{T_p} [w(t).w(t-\Delta_n)]dt$$

The first integral denotes the desired signal *(DS)* where a "ref" pulse overlaps with a "data" pulse. Since pulses are uncorrelated, it is important to overlap the "ref" pulse of the user of interest with the received signals' "data" pulse to achieve maximum autocorrelation to achieve more reliable signal detection in noisy environments. For this reason, the interference of other users "ref" pulse to the user of interest's "ref" pulse is removed by Gram-Schmidt Process to preserve the shape of the original "ref" pulses used for each channel. Considering the reference part of received signal from (1), we have

$$ref_n(t) = \sum_{m=1}^{M} P_n(t-(m-1)T) \qquad (14)$$

Where $ref_n(t)$ is the reference pulse for n^{th} user and M is the maximum number of bits. The sum of reference pulses for all users at the receiver is

$$ref(t) = ref_1(t) + ref_2(t) + ... + ref_N(t) \qquad (15)$$

Gram-Schmidt process removes the projection of the unwanted "*ref*" pulses from the desired "*ref*" pulse for the user of interest as shown below

$$ref_n(t) = ref(t) - \int_{j=1, j\neq n}^{N} \frac{ref^*(t).ref_j(t)}{\|ref_j(t)\|}.ref_j(t)\,dt \qquad (16)$$
$$n = \{1,2,..., N\}$$

The first integral in (13) represents the desired signal and the second, third and fourth integrals in represent performance degradation to the received signal based on signal-on-signal interference *(I_{ss})*, signal-on-noise interference *(I_{sn})*, and noise-on-noise interference *(I_{nn})* respectively. Because of the minimal correlation between the mutually orthogonal chirp pulses used in our design, the performance degradation is mainly due to *(I_{nn})* and *(I_{sn})*. The integrals in equation (13) are used in next section to derive the theoretical representation of the systems' probability of error.

4. MPMD PERFORMANCE

In this section the performance of the MPMD system is analyzed based on multiple delays and multiple autocorrelation sampling points. All bit error rate simulations in this section were carried out with total number of 100,000 bits per data point. Fig. 3 illustrates the BER versus SNR for a variety of delay separations (D_n) between ten users in various SNR levels.

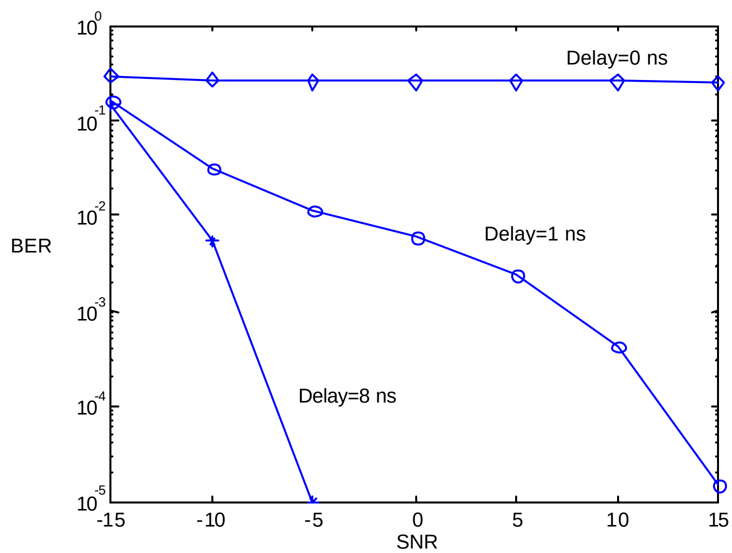


Figure 3: BER versus SNR based on delay separation between users for a 10-user system with 1 Sampling point (k=1)

As shown in Fig.3 the worst BER occurs when there is no separation between the users. However, a minimal separation between users shows a considerable BER performance. Fig.4 represents the performance improvement by increasing the number of sampling points (K) in the received signal's ACF. Higher number of sampling points in ACF of the received signal provides more accuracy in the match filtering process and results in BER improvements of the system.

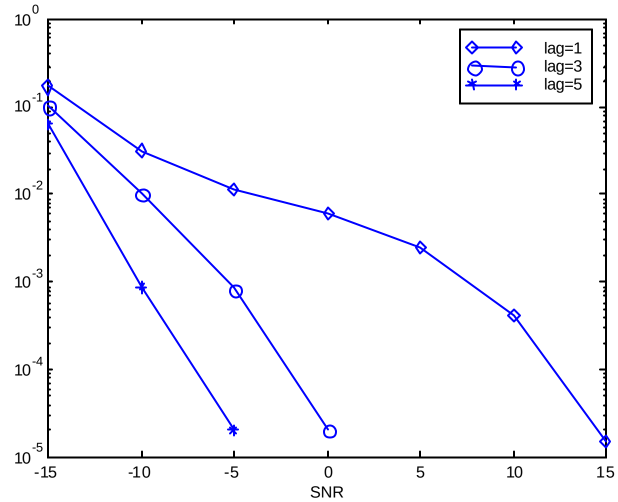


Figure 4: BER Versus Autocorrelation Sampling Points *(K)* for 10 Users at with 1ns Separation Between Users

The results from Fig.3 and 4, proves that channel capacity can be increased significantly compared to the time hopping modulations that require a large separation between users. This modulation scheme also is very effective in multipath environments. Since the same pulse is sent twice through a channel where both pulses are distorted the same way by multipath. Therefore the autocorrelation of the distorted "ref" pulse and distorted "data" pulse can still recover the transmitted signal. The performance of the proposed system at the presence of multipath will be the subject of a subsequent publication by the authors.

As mentioned earlier, I_{ss} has minimal effect on the system's performance degradation due to the orthogonality of pulses in MPMD system. Therefore, receiver's BER performance depends mainly on the I_{sn} and I_{nn} components of (13). The theoretical value of the system's probability of error can be calculated as a BPSK bit error rate since the decision on the output of the integrator is similar to the demodulation of an antipodal signal. Note that the theoretical limit for BPSK bit error rate is obtained by [14]

$$P_e = \frac{1}{2} Q(\sqrt{SNR}) \tag{17}$$

Where the function *Q(.)* is defined as the complementary error function, i.e.,

$$Q(x) = \frac{2}{\sqrt{p}} \int_x^{\infty} e^{-t^2} dt \tag{18}$$

The Signal-to-Noise Ratio is

$$SNR = \frac{DS^2}{Var(I_{sn} + I_{nn})} = \frac{DS^2}{E\{I_{sn}{}^2\} + E\{I_{nn}{}^2\}} \tag{19}$$

The desired signal *(DS)* is calculated by taking the delays out of the first integral in (13) and integrating over pulse width (T_p)

$$DS = [\, l_1 . E_{P_n} + \sum_{k=2}^{K} l_k . R_{P_n P_n} . d_{nk} \,] \tag{20}$$

In this equation $R_{P_n P_n} . d_{nk}$ represents the autocorrelation values of each users pulse *(P_n)* in *K, non-zero* lags. I_{sn} and I_{nn} in (19) can be calculated the same way as *DS* using third and fourth integrals in (13)

$$I_{sn} = [\, l_1 . E_{P_n} . N_0 + R_{P_n P_n} . N_0 \sum_{k=2}^{K} l_k . d_{nk} \,] \tag{21}$$

$$I_{nn} = B . T_p . \frac{N_0{}^2}{4} \sum_{k=1}^{K} l_k \tag{22}$$

In these terms B represents the bandwidth of a low pass filter used on noise. Since integration of the fourth term in (13) over the pulse width (T_p) is equivalent of passing the noise-on-noise interference *(I_{nn})* through a low pass filter of bandwidth $1/T_p$ and multiplying its amplitude by T_p. Therefore the total probability of error for a single user case (n=1) using (19) is

$$P_e = \frac{1}{2} Q\left(\sqrt{ \frac{ [\, l_1 . E_{P_n} + \sum_{k=2}^{K} l_k . R_{P_n P_n} . d_{nk} \,]^2 }{ l_1 . E_{P_n} . N_0 + N_0 . R_{P_n P_n} \sum_{k=2}^{K} l_k . d_{nk} + B . T_p . \frac{N_0{}^2}{4} \sum_{k=1}^{K} l_k } } \right) \tag{23}$$

A detailed and step-by-step derivation of *SNR* for the system can be found at Appendix A.

In Fig 5 the BER in an AWGN channel for a single user is compared against the analytical bit error rate as given by (23).

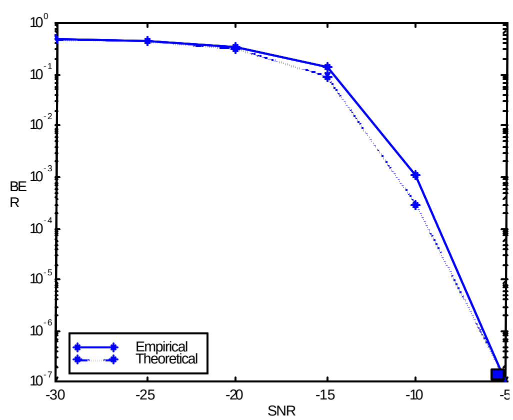


Figure 5: Comparison of Theoretical Versus Empirical BER Values for a Single User Detection System at K=1; (■) indicates that BER = 0 for both curves at –5 dB SNR.

The minimal discrepancy between the two graphs is due to the fact that signal spectrum and noise spectrum are approximations to frequency band, and in practice the stopband noise of the filter can leak to the passband. While in theoretical calculations, it is assumed that the filter is perfect and there is no leakage of stopband noise to passband. At very low SNR (-30 dB to –20dB) as shown in Fig. 5, there is no mismatch between the values since the system is saturated with noise and the leakage of noise does not make the results worse. As SNR increases, signal dominates the noise and theoretical and empirical values match again.

5. CONCLUSIONS

The new MPMD multiple access method proposed in this paper provides significant improvement in bit error rate performance of a UWB multiple access system in low SNR channels. The analysis reveals that the use of multiple uncorrelated pulses minimizes the MAI and allows the system to increase the transmission rate. The time delays used to separate the users can be a fraction of the pulse duration for each channel providing acceptable BERs. We also showed that the number of sampling points of the autocorrelation function plays an important role to increase the performance of a UWB multiple access system. The BER results show good agreement between theory and simulations. The results show that the proposed method is efficient as a multiple access UWB technique in high interference environments. Future work will include the use of MPMD method in heavy multipath channels.

APPENDIX

SIGNAL-TO-NOISE RATIO CALCULATION

In order to derive the receivers' Signal-to-Noise Ratio (SNR) we start with a simple case of single user detection system with *lag zero* (k =1) sampling as shown in Fig. A.1.

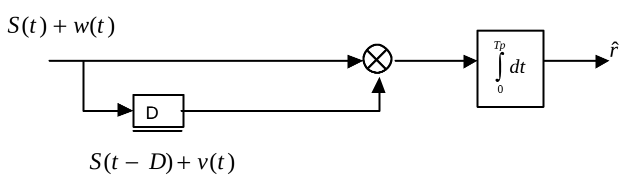


Figure A.1: Single User Detector System with k =1

Where w(t) and v(t) are different samples of white noise, independent and uncorrelated with each other. As shown in Fig. A.1, the received signal can be calculated as:

$$\hat{r}=\int_0^{T_p}[S(t)+w(t)].[S(t-D)+v(t)]dt \tag{A.1}$$

To find the systems' SNR we should determine the signal and noise components of the Variance of $\hat{r}$, or $E\{\hat{r}^2\}$. The multiplication in A.1 results in the following 4 integrals:

$$\hat{r}=\int_0^{T_p}[S(t)+S(t-D)]dt+\int_0^{T_p}[S(t)w(t)]dt+$$
$$\int_0^{T_p}[S(t-D)v(t)]dt+\int_0^{T_p}[w(t)v(t)]dt$$

(A.2)

Similarly

$$\hat{r}^2 = [R_{SS}(D) + \int_0^{T_p} S(t_1)\,w(t_1)dt_1 + \int_0^{T_p} S(t_1 - D)\,w(t_1)dt_1 + \int_0^{T_p} w(t_1)\,v(t_1)dt_1].[R_{SS}(D) + \int_0^{T_p} S(t_2)\,w(t_2)dt_2 + \int_0^{T_p} S(t_2 - D)w(t_2)dt_2 + \int_0^{T_p} w(t_2)\,v(t_2)dt_2]$$

(A.3)

The above multiplication results in 16 terms $[I_1 + I_2 + . \quad . \quad . + I_{16}]$ and Variance can be calculated by taking the $E\{.\}$ operation over each of the 16 terms:

$$E\{\hat{r}^2\} = E\{[I_1 + I_2 + . \quad . \quad . + I_{16}]\} \tag{A.4}$$

$$E\{I_1\} = R_{SS}{}^2(D) \tag{A.5}$$

Note $E\{I_1\} = E_p{}^2$ (Signal energy) for lag zero or main lobe of ACF.

$$E\{I_2\} = R_{SS}(D).\int_0^{T_p} S(t_2)E\{v(t_2)\}\,dt_2 = 0 \tag{A.6}$$

Similarly $E\{I_3\}$, $E\{I_4\}$, $E\{I_5\}$, $E\{I_7\}$, $E\{I_8\}$, $E\{I_9\}$, $E\{I_{10}\}$, $E\{I_{12}\}$, $E\{I_{13}\}$, $E\{I_{14}\}$, $E\{I_{15}\}$ result in zero because noise is assumed to be a zero mean random process.

$$E\{I_6\} = \int_0^{T_p} S(t_1)dt_1.\int_0^{T_p} S(t_2)E\{v(t_1)v(t_2)\}\,dt_2$$
$$= \frac{N_0}{2}\int_0^{T_p} S(t_1).S(t_2)dt_1 = \frac{N_0}{2}.E_p$$

(A.7)

$$E\{I_{11}\} = \int_0^{T_p} S(t_1 - D)dt_1.\int_0^{T_p} S(t_2 - D)E\{w(t_1)v(t_2)\}\,dt_2$$
$$= \frac{N_0}{2}\int_0^{T_p} S(t_1 - D)S(t_2 - D)dt_1 \approx \frac{N_0}{2}.R_{ss}(D) \approx \frac{N_0}{2}.E_p$$

(A.8)

$$E\{I_{16}\} = \int_0^{T_p} dt_1.\int_0^{T_p} E\{w(t_1)w(t_2)\,v(t_1)v(t_2)\}\,dt_2 = \frac{N_0{}^2}{4} \tag{A.9}$$

The results from A.9 are based on the fourth moment of jointly Gaussian variables. Hence the Variance of $\hat{r}$ is

$$E\{\hat{r}^2\} = [R_{ss}{}^2(D) + \frac{N_0}{2}.E_p + \frac{N_0}{2}.E_p + \frac{N_0{}^2}{4}] \tag{A.10}$$

Since integrating the fourth integral in A.2 over the pulse width (T_p) is equivalent to passing the noise-on-noise interference through a low pass filter of bandwidth $1/T_p$ and multiplying its amplitude by T_p the (I_{nn}) term would be

$$I_{nn} = \frac{N_0{}^2}{4}.B.T_p \tag{A.11}$$

Where B is the bandwidth of low pass filter used on noise, since bandwidth of noise is much larger than $1/T_p$. The signal-on-noise interference (I_{ss}) is

$$I_{sn} = N_0 E_p \tag{A.12}$$

and the desired signal is

$$DS^2 = R_{ss}{}^2(D) = E_p{}^2 \qquad (\,forlagzero) \tag{A.13}$$

Which results in the following SNR for a single user, lag zero case

$$SNR = \frac{E_p{}^2}{N_0 E_p + \frac{N_0{}^2}{4}.B.T_p} \tag{A.14}$$

Based on the above basic derivations, I_{nn}, I_{sn}, DS^2 can be derived based on expansion of A.11, A.12, and A.13 for more lags (k >1) and more number of users respectively.

$$I_{nn} = B.T_p.\frac{N_0{}^2}{4}\sum_{k=1}^{K} l_k \tag{A.15}$$

$$I_{sn} = [\, l_1.E_{P_n}.N_0 + R_{P_nP_n}.N_0 \sum_{k=2}^{K} l_k.d_{nk}\,] \tag{A.16}$$

$$DS^2 = [\, l_1.E_{P_n} + \sum_{k=2}^{K} l_k.R_{P_nP_n}.d_{nk}\,]^2 \tag{A.17}$$

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